Risk Aversion in the Nash Bargaining Problem with Uncertainty

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Abstract

We apply the aggregation property of Identical Shape Harmonic Absolute Risk Aversion (ISHARA) utility functions to analyze the comparative statics properties of a bargaining model with uncertainty. We identify sufficient and necessary conditions under which an increase in one's degree of risk aversion benefits/hurts one's opponent. We apply our model to analyze the problems of bargaining over insurance contracts and bargaining over incentive contracts.

Keywords Bargaining, the Nash Solution, ISHARA Preference, Risk Aversion

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1 Introduction

In many real-world situations, transactions take place through bargaining. Labour markets in most western economies are characterized by collective agreements negotiated between unions and firms; non-unionized workers' salaries are commonly set by individual negotiation, which is most clearly the case for managerial compensation. Firms negotiate over how to split the profits from a joint venture; buyers and sellers bargain over the price of a product; and, in the insurance market, the insurer and his client negotiate over the insurance contract (Kihlstrom and Roth 1982).

Almost all of the bargaining situations mentioned above have something in common—they involve uncertainty (White 2008). Individuals do not know whether an accident will happen when they are bargaining over the insurance contract; the firm and the manager have no idea whether the manager will perform well when they are deciding the manager's compensation package; producers and retailers are uncertain about the exact demand when they are setting wholesale prices.

Compared to the well-understood situation of bargaining with a deterministic outcome, bargaining with a risky outcome is much more difficult to study, especially with regard to the analysis of comparative statistics. For example, a frequently cited proposition in the deterministic bargaining literature asserts that an increase in one's degree of risk aversion improves the welfare of one's opponent. Intuitively, the subjective possibility of strategically reaching disagreement and its costly consequence makes risk aversion disadvantageous in bargaining (Kannai, 1977; Roth, 1979; Kihlstrom, Roth and Schmeidler, 1981; Sobel, 1981). However, it may fail in the case of a risky outcome and riskless disagreement (Roth and Rothblum, 1982), as well as in the case of a risky outcome and risky disagreement (Safra, Zhou and Zilcha, 1990).

The complexity of the analysis of the comparative statics properties of bargaining models with risky outcome and risky disagreement impedes the application of such models. This paper provides a simple method to deal with this situation by focusing on the Identical Shape Harmonic Absolute Risk Aversion (ISHARA) utility functions. The ISHARA assumption — under which risk tolerances are linear in income with identical slope — implies an aggregation property: The sum of the certainty equivalents for the two bargainers is independent of the sharing rule that is used as long as the sharing rule is "efficient". Therefore, the model of bargaining over a risky outcome can be reduced to a problem of bargaining over a certainty equivalent — a riskless outcome.

This transformation allows us to disengage two effects regarding an increase in one's degree of risk aversion: the bargaining power effect and the net surplus effect. On the one hand, a more risk-averse bargainer has weak bargaining power and hence his opponent benefits. On the other hand, an increase in one's degree of risk aversion changes the size of the net certainty equivalent — the total certainty equivalent of agreement minus the sum of the certainty equivalents of the two bargainers' disagreements — that the two bargainers are bargaining over. This will benefit (resp. hurts) his opponent if the size is increased (resp. reduced). Consequently, the welfare of one's opponent will be increased as long as an increase in one's degree of risk aversion increases the net certainty equivalent. The welfare of one's opponent will be reduced if an increase in one's degree of risk aversion significantly reduces the net certainty equivalent.

We then apply our model to analyze two situations: bargaining over an insurance contract and bargaining over an incentive contract.

Determining the insurance contract through bargaining between the insurer and the client is justified if neither of them is small. Kihlstrom and Roth (1982) study such a problem with very general utility function and show that an insurer always benefits as the client becomes more risk-averse. However, they analyze only the case of the risk-neutral insurer, although they note that the assumption of the insurer's risk neutrality cannot be justified in some interesting situations. They argue that subsequent work would require an extension of their results to the case of the risk-averse insurer. That is precisely the work of this paper. The simple transformation allows us to easily check that their results are still valid in the case of the risk-averse insurer.

Another application considers the problem of bargaining over an incentive contract. Standard principal-agent models always assume that the principal offers a "take-it-or-leave-it" offer. However, as we have already argued, it is common that, in real-life situations, both parties hold some bargaining power. In models of bargaining over incentive contracts, economists show that the distribution of bargaining power between principal and agent has real effects (Pitchford 1998, Balkenborg 2001, Schmitz 2005, Demougin and Helm 2006, Demougin and Helm 2009, Dittrich and Städter 2011, Yao 2012). However, that literature considers only the case with a risk-neutral principal and a wealth-constrained risk-neutral agent (or a risk-neutral agent with limited liability). This paper complements this literature by considering bargaining between a risk-neutral principal and a risk-averse agent à la Holmstrom and Milgrom (1987).

We show that the bargaining model predicts the same power of incentives and total surplus as does the model in which the principal makes a "take-it-or-leave-it" offer. However, the principal's preference over the agent's degree of risk aversion is quite different. If the principal holds all the bargaining power, he is worse off when the agent becomes more risk-averse, as providing incentive becomes more costly and, hence, the total surplus is reduced. When the contract is determined through bargaining, this result may not hold because an increase in the agent's degree of risk aversion has two effects. On the one hand, it reduces the total surplus; on the other hand, it also reduces the agent's bargaining power. For a sufficiently riskless production process, the first effect is dominated by the second one, leading to a higher utility for the principal.

The paper is organized as follows. Section 2 lays out the basic model. Section 3 provides the solution. Section 4 applies the basic model to the problems of bargaining over insurance and incentive contract. We conclude in Section 5.

2 The Nash Bargaining Game

Two bargainers are bargaining over a risky outcome $\tilde{Y}$. Bargainer i has vNM utility function $u_i(w) : [0, \infty) \to \mathbb{R}$, $i = 1, 2$. The bargaining game is defined by a pair (S, d), where $S = \{(Eu_1(s(\tilde{Y})), Eu_2(\tilde{Y} - s(\tilde{Y})))|0 \leq s(Y) \leq Y\}$ is the set of (unanimously agreed) feasible expected utility payoffs to the bargainers, $d = (Eu_1(\tilde{y}_1), Eu_2(\tilde{y}_2))$ is the disagreement point, $s(Y)$ is the risk sharing rule that maps each realized value of $\tilde{Y}$ to bargainer 1's individual share, and $\tilde{y}_i$ is bargainer i's disagreement payoff.

We allow $\tilde{Y}$ and $\tilde{y}_i$s to be degenerated random variables, i.e., riskless variables. If none of $\tilde{Y}$ and $\tilde{y}_i$s is degenerated, we are in the case of risky agreement and risky disagreement; if $\tilde{Y}$ is degenerated, we are in the case of riskless agreement and risky disagreement; if $\tilde{y}_i$s are degenerated, we are in the case of risky agreement and riskless disagreement. Here, $\tilde{Y}$ and $\tilde{y}_i$s can be independent or correlated.

The Nash solution will specify risk-sharing rules $\hat{s}(Y)$, which solves the following problem:

$$\textbf{P1} \max_{s(Y)} \Big((Eu_1(s(\tilde{Y})) - Eu_1(\tilde{y}_1)) \cdot (Eu_2(\tilde{Y} - s(\tilde{Y})) - Eu_2(\tilde{y}_2)) \Big),$$

and yields the bargaining outcomes $F_1(S, d) = Eu(\hat{s}(\tilde{Y}))$, $F_2(S, d) = Eu(\tilde{Y} - \hat{s}(\tilde{Y}))$ for bargainer 1 and 2 respectively.

The question that is central to this paper is: will bargainer 1 be better off if bargainer 2 becomes more risk-averse, i.e., his utility function becomes $v_2(c)$, with $\frac{-v_2''(c)}{v_2'(c)} > \frac{-u_2''(c)}{u_2'(c)}$?

3 Solution with ISHARA utility functions

Problem **P1** concerning risk-sharing rules is not easy to solve. We hence focus on the case of Identical Shape Harmonic Absolute Risk Aversion (ISHARA) utility functions, because this kind of utility functions avoids the problem that the total surplus changes with the risk-sharing rule. Following Schulhofer-Wohl (2006), we give the following definition of ISHARA:

Definition 1 *The two bargainers have ISHARA preferences if their utility functions are given by* $u_i(c) = \frac{(c-\theta_i)^{1-\sigma}}{1-\sigma}$*,* $i = 1, 2$*, where* $\sigma \geq 0$ *is common to both bargainers and* θ_i *is bargainer i's individual parameter.*

Notice that the constant absolute risk aversion is a special case in the limit as σ goes to infinity with θ/σ fixed.

Aggregation Property. It is well known that with ISHARA utility functions, the Pareto frontier in the monetary-equivalent space is a straight line and the monetary value of the joint pie is distribution-free, i.e., the sum of the two bargainers' certainty equivalents is *constant* for any efficient risk sharing rule and *does not* depend on the weights given to the bargainers (see Schulhofer-Wohl 2006 for a proof). We call this property the **aggregation property**. Thus the Nash solution to bargaining with risky outcomes and risky disagreement points can be viewed as the division of a fixed amount of certainty equivalent between two risk-averse bargainers. Formally, denote $C_1 = u_1^{-1}\left(Eu_1\left(s\left(\tilde{Y}\right)\right)\right)$ as bargainer 1's certainty equivalent, $C_2 = u_2^{-1}\left(Eu_2\left(\tilde{Y} - s\left(\tilde{Y}\right)\right)\right)$ as bargainer 2's certainty equivalent. Then, for any efficient risk sharing rule $s\left(Y\right)$, we must have $C_1 + C_2 \equiv C$, where C is constant, representing the total certainty equivalent bargained over by the two bargainers.

Denote $C_1^d \triangleq u_1^{-1}(Eu_1(\tilde{y}_1)), C_2^d \triangleq u_2^{-1}(Eu_2(\tilde{y}_2))$ as the two bargainers' disagreement payoffs in monetary terms. The net surplus in terms of certainty equivalent is $NC = C - \left(C_1^d + C_2^d\right) > 0$. Henceforth, whenever we say "the size of the pie", we refer to the net surplus NC.

Since the Nash solution is Pareto optimal and satisfies the axiom of independence of irrelevant alternatives, we can restrict our attention to the Pareto frontier which, under this transformation, is given by $S^p = \{(u_1(C_1), u_2(C_2)) \,|C_1 \geq 0, C_2 \geq 0, C_1 + C_2 = C\}$[1]. Because each bargainer should obtain at least his

[1] Independence of irrelevant alternatives means that the solution to the bargaining problem does not change if the utility possibilities set is unfavorably altered such that the disagreement point is unchanged and the original solution remains feasible. That is, if (S, d) and (S', d) are bargaining problems and $S' \subset S$, and the solution of (S, d) also belongs to S', then the two bargaining problems have the same solution.

disagreement utility, we can further restrict our attention to $\tilde{S}^p = \{(u_1(C_1), u_2(C_2)) \,|C_1 \geq C_1^d, C_2 \geq C_2^d, C_1 + C_2 = C\}$, which, using the expression of NC, can be rewritten as $\tilde{S}^p = \{\left(u_1(C_1^d + x), u_2(C_2^d + NC - x)\right) |0 \leq x \leq NC\}$. It can be easily proved that there exists a unique Nash solution on $\tilde{S}^P$, and the solution (in the certainty-equivalent space) can be obtained from the following maximization problem:

$$\mathbf{P2} \max_{0 \leq x \leq NC} \left(\left(u_1\left(C_1^d + x\right) - u_1\left(C_1^d\right)\right) \cdot \left(u_2\left(C_2^d + NC - x\right) - u_2\left(C_2^d\right)\right)\right).$$

Thus, we have transformed the bargaining model with risky agreement and risky disagreement into a bargaining model with riskless agreement and riskless disagreement. Denote $w_1(c) = u_1\left(C_1^d + c\right)$ and $w_2(c) = u_2\left(C_2^d + c\right)$. The above bargaining problem can be viewed as two bargainers, whose utility functions are $w_1(c)$ and $w_2(c)$ with disagreement payoffs zero, are bargaining over a riskless pie NC.

$$\mathbf{P2'} \max_{0 \leq x \leq NC} \left(\left(w_1(x) - w_1(0)\right) \cdot \left(w_2(NC - x) - w_2(0)\right)\right).$$

Denote the solution as $x^{\#}$. The F.O.C. with respect to x gives:

$$w_1'\left(x^{\#}\right)\left[w_2\left(NC - x^{\#}\right) - w_2(0)\right] - w_2'\left(NC - x^{\#}\right)\left[w_1\left(x^{\#}\right) - w_1(0)\right] = 0, \quad (1)$$

which, after rearranging, yields:

$$\frac{w_1\left(x^{\#}\right) - w_1(0)}{w_1'\left(x^{\#}\right)} = \frac{w_2\left(NC - x^{\#}\right) - w_2(0)}{w_2'\left(NC - x^{\#}\right)}, \quad (2)$$

i.e., the ratio of each bargainer's net share of the pie in terms of expected utility to marginal utility should be equal.

Now consider the effect of replacing bargainer 2's preference with a more risk-averse utility function v_2. The increase in risk aversion has two effects. First, it reduces the sum of the certainty equivalent. Denote the reduced amount as $\Delta C = C - C^*$, where we use the superscript $*$ to denote the corresponding variables in the new bargaining game between bargainer u_1 and bargainer v_2. Second, it also reduces the disagreement certainty equivalent of bargainer 2. Denote the reduced amount as $\Delta C_2^d = C_2^d - C_2^{d*}$. The reduced amount of the size of the pie (the net surplus) is hence $\Delta NC = NC - NC^* = \Delta C - \Delta C_2^d$. When $\Delta NC > 0$, the size of the pie decreases after the replacement; when $\Delta NC < 0$, the size of the pie increases after the replacement. The solution x^* solves the following problem:

$$\mathbf{P3} \max_{0 \leq x \leq NC^*} \left(\left(w_1(x) - w_1(0)\right) \cdot \left(w_2^*\left(NC^* - x\right) - w_2^*(0)\right)\right),$$

where $NC^* = C^* - C_1^d - C_2^{d*}$ is the net surplus in the bargaining game between bargainer u_1 and bargainer v_2, and $w_2^*(c) = v_2\left(C_2^{d*} + c\right)$. The bargaining game can be viewed as two bargainers, whose utility functions are $w_1(c)$ and $w_2^*(c)$, and who are bargaining over a riskless pie NC^*, with disagreement payoffs zero. Similarly, as in solving $\mathbf{P2}'$, the F.O.C. yields

$$\frac{w_1(x^*) - w_1(0)}{w_1'(x^*)} = \frac{w_2^*(NC^* - x^*) - w_2^*(0)}{w_2^{*\prime}(NC^* - x^*)}. \tag{3}$$

Bargainer 1 prefers to bargain with bargainer v_2 rather than with bargainer u_2 if $x^* \geq x^\#$, which is the case iff

$$w_1'(x^*)\left[w_2(NC - x^*) - w_2(0)\right] - w_2'(NC - x^*)\left[w_1(x^*) - w_1(0)\right] \leq 0,$$

which, after rearranging, yields

$$\frac{w_2(NC - x^*) - w_2(0)}{w_2'(NC - x^*)} \leq \frac{w_1(x^*) - w_1(0)}{w_1'(x^*)}.$$

Substitute equation (3) into the above inequality, and we get the necessary and sufficient condition of $x^* \geq x^\#$:

$$\frac{w_2(NC - x^*) - w_2(0)}{w_2'(NC - x^*)} \leq \frac{w_2^*(NC^* - x^*) - w_2^*(0)}{w_2^{*\prime}(NC^* - x^*)}, \tag{4}$$

which can be rewritten as:

$$\begin{aligned}&\left[\frac{w_2(NC - x^*) - w_2(0)}{w_2'(NC - x^*)} - \frac{w_2^*(NC - x^*) - w_2^*(0)}{w_2^{*\prime}(NC - x^*)}\right] \\ &+\left[\frac{w_2^*(NC - x^*) - w_2^*(0)}{w_2^{*\prime}(NC - x^*)} - \frac{w_2^*(NC^* - x^*) - w_2^*(0)}{w_2^{*\prime}(NC^* - x^*)}\right] \leq 0.\end{aligned} \tag{5}$$

An increase in one's degree of risk aversion has two effects on one's opponent's welfare. First, when one becomes more risk-averse, one's bargaining power will change. The term in the first square bracket reflects this bargaining power effect, because it keeps the net certainty equivalent unchanged. Second, the net surplus also changes as one becomes more risk-averse. This net surplus effect is reflected by the terms in the second square bracket. The following lemma tells us that the sign of the first bracket is always negative.

Lemma 1 $\frac{w_2(NC-x^*)-w_2(0)}{w_2'(NC-x^*)} - \frac{w_2^*(NC-x^*)-w_2^*(0)}{w_2^{*\prime}(NC-x^*)} \leq 0.$

Proof. *Denote $\delta = NC - x^*$. The inequality is equivalent to*

$$\int_0^\delta \frac{w_2'(c)}{w_2'(\delta)}dc \leq \int_0^\delta \frac{w_2^{*\prime}(c)}{w_2^{*\prime}(\delta)}dc$$

$$\Leftarrow \frac{w_2'(c)}{w_2'(\delta)} \leq \frac{w_2^{*\prime}(c)}{w_2^{*\prime}(\delta)}, \forall c < \delta$$

$$\Leftrightarrow \frac{w_2'(c)}{w_2^{*\prime}(c)} \leq \frac{w_2'(\delta)}{w_2^{*\prime}(\delta)}, \forall c < \delta,$$

which holds if $\frac{w_2'(c)}{w_2^{*\prime}(c)}$ is increasing in c.

$$\frac{\partial}{\partial c}\frac{w_2'(c)}{w_2^{*\prime}(c)} = \frac{w_2''(c)\, w_2^{*\prime}(c) - w_2'(c)\, w_2^{*\prime\prime}(c)}{w_2^{*\prime}(c)^2} > 0,$$

$$\Leftrightarrow -\frac{w_2''(c)}{w_2'(c)} < -\frac{w_2^{*\prime\prime}(c)}{w_2^{*\prime}(c)}.$$

Because bargainer v_2 is more risk-averse than bargainer u_2, we have $-\frac{w_2''(c)}{w_2'(c)} = -\frac{u_2''\left(C_2^d+c\right)}{u_2'\left(C_2^d+c\right)} < -\frac{v_2''\left(C_2^d+c\right)}{v_2'\left(C_2^d+c\right)}$. Moreover, our assumption that $\sigma \geq 0$ implies v_2 exhibits Decreasing Absolute Risk Aversion property, and hence $-\frac{v_2''\left(C_2^d+c\right)}{v_2'\left(C_2^d+c\right)} < -\frac{v_2''\left(C_2^{d*}+c\right)}{v_2'\left(C_2^{d*}+c\right)} = -\frac{w_2^{*\prime\prime}(c)}{w_2^{*\prime}(c)}$ due to $C_2^{d*} < C_2^d$. ■

Notice that equation (4) is equivalent to Lemma 1, given that $NC = NC^*$. The above lemma states that an increase in bargainer 2's degree of risk aversion, if it doesn't affect the net bargaining surplus, i.e., $\Delta C = \Delta C_2^d$, will make bargainer 1 better off. This result is consistent with the prevailing predictions on the Nash solution with risk-averse bargainers: risk aversion benefits one's opponent (Kihlstrom, Roth, and Schmeidler, 1981; Roth, 1979, among others). Disagreement has costly consequences, and the desire to avoid the risk of disagreement is reflected in the final bargaining outcome. A more risk-averse bargainer has a stronger desire to avoid such risk, and hence is willing to give up more share during the bargaining in order to facilitate reaching an agreement.

Lemma 2 *$\frac{w_2^*(NC-x^*)-w_2^*(0)}{w_2^{*\prime}(NC-x^*)}$ is increasing in NC.*

Proof. *The result is straightforward by noticing that $w_2^*(NC - x^*) - w_2^*(0)$ is increasing in NC and that $w_2^{*\prime}(NC - x^*)$ is decreasing in NC.* ■

Thus, the term in the second square bracket of (5), reflecting the net surplus effect, is negative if $NC^* > NC$. It states an intuitive result: bargainer 1 will be better off as the size of the pie increases.

Combining lemma 1 and lemma 2 yields the following proposition:

Proposition 1 *An increase in one's degree of risk aversion benefits one's opponent if the net certainty equivalent increases. It hurts one's opponent only if the net certainty equivalent decreases significantly, i.e., when it outweighs the opponent's benefit from the increase of relative bargaining power.*

As bargainer 2 becomes more risk-averse, the total certainty equivalent will decrease significantly when the agreement income $\tilde{Y}$ is highly risky. Bargainer 2's total certainty equivalent will decrease significantly when his/her disagreement income $\tilde{y}_2$ is highly risky. The net certainty equivalent is more likely to increase when $\tilde{Y}$ is not risky and $\tilde{y}_2$ is highly risky; while it will decrease when $\tilde{Y}$ is highly risky and $\tilde{y}_2$ is not risky. In the case of riskless agreement and risky disagreement, the total certainty equivalent does not change, while bargainer 2's certainty equivalent of disagreement decreases as he/she becomes more risk-averse. Thus the net surplus increases and hence benefits bargainer 1. In the case of risky agreement and riskless disagreement, the net certainty equivalent decreases and bargainer 1 may become worse off if $\tilde{Y}$ is very risky. In the case of risky agreement and risky disagreement, whether the net certainty equivalent increases or decreases depends on the relative riskiness of $\tilde{Y}$ and $\tilde{y}_2$.

Finally, the change in the size of the pie also depends on the relative degree of risk aversion between the two bargainers. If bargainer 1 is much less risk-averse than bargainer 2, then bargainer 1 bears most of the risk. Thus, an increase in bargainer 2's degree of risk aversion would not change the total certainty equivalent very much. In the extreme case, where bargainer 1 is risk-neutral, the total certainty equivalent remains unchanged. Therefore, the size of the pie increases as bargainer 2's certainty equivalent of disagreement decreases. Similar arguments tell us that the size of the pie will be reduced if bargainer 2's degree of risk aversion is much less than that of bargainer 1. We summarize these results as a corollary of Proposition 1.

Corollary 1 *(1) With riskless agreement, an increase in a bargainer's degree of risk aversion always increases his/her opponent's welfare.*

(2) With risky agreement, when the degree of a bargainer's risk aversion increases, its impact on his/her opponent's welfare is ambiguous. Specifically, an increase in a bargainer's degree of risk aversion is more likely to decrease (resp. increase) his/her opponent's welfare in the following three situations, ceteris paribus:

(2-a) the agreement is highly (resp. less) risky;

(2-b) the bargainer's disagreement is less (resp. highly) risky;

(2-c) the bargainer's degree of risk aversion is much less (resp. higher) than that of his/her opponent.

We illustrate the above proposition with the following example.

Example 1 *Consider the case where two bargainers have CARA utility function* $u_i(c) = \frac{1-\exp(-r_i c)}{r_i}$, $i = 1,2$. *Assume,* $\tilde{Y} \sim N(\mu, \sigma^2)$, $\tilde{y}_i \sim N(\mu_i, \sigma_i^2)$. *The specific assumption allow us to write* $C = \mu - \frac{R}{2}\sigma^2$, $C_1^d = \mu_1 - \frac{r_1}{2}\sigma_1^2$ *and* $C_2^d = \mu_2 - \frac{r_2}{2}\sigma_2^2$. *The net certainty equivalent is given by* $NC = C - C_1^d - C_2^d$.

An increases in r_2 *will benefit (resp. hurts) bargainer 1 if* $f(r_2, NC) = \frac{u_2\left(C_2^d+NC-x\right)-u_2\left(C_2^d\right)}{u_2'\left(C_2^d+NC-x\right)}$ *is increasing (resp. decreasing) in* r_2. *Denote* $\delta = NC - x$.

$$\begin{aligned}
\frac{d}{dr_2} f(r_2, NC) &= \frac{\partial}{\partial r_2} f(r_2, NC) + \frac{\partial}{\partial NC} f(r_2, NC) \frac{\partial NC}{\partial r_2} \\
&= \frac{1}{r_2}\left(1 + r_2\delta e^{r_2\delta} - e^{r_2\delta}\right) + e^{r_2\delta}\left[\frac{1}{2}\left(\sigma_2^2 - \frac{\partial R}{\partial r_2}\sigma^2\right)\right] \\
&= \frac{1}{r_2}\left(1 + r_2\delta e^{r_2\delta} - e^{r_2\delta}\right) + e^{r_2\delta}\left[\frac{1}{2}\left(\sigma_2^2 - \frac{r_1^2}{(r_1+r_2)^2}\sigma^2\right)\right].
\end{aligned}$$

It is easy to prove that $1 + r_2\delta e^{r_2\delta} - e^{r_2\delta} > 0$. *Therefore, we have* $\frac{d}{dr_2} f(r_2, NC) > 0$ *when* $\sigma_2^2 - \frac{r_1^2}{(r_1+r_2)^2}\sigma^2 > 0$, *which is more likely to be the case if* σ_2 *is large,* σ^2 *is small and that* r_2 *is much larger than* r_1.

That $\frac{d}{dr_2} f(r_2, NC) < 0$ *occurs only if* $\sigma_2^2 - \frac{r_1^2}{(r_1+r_2)^2}\sigma^2 < 0$. *Consider the case with riskless disagreement where* $\sigma_2^2 = 0$. $\frac{d}{dr_2} f(r_2, NC) < 0$ *will be the case if* $\sigma^2 > \hat{\sigma}^2$, *with* $\hat{\sigma}^2 = \frac{2(1+r_2/r_1)^2\left(1+r_2\delta e^{r_2\delta}-e^{r_2\delta}\right)}{r_2 e^{r_2\delta}}$. *Notice that* $\hat{\sigma}^2$ *is increasing in* r_2/r_1, *which means that* $\frac{d}{dr_2} f(r_2, NC) < 0$ *is easier to be satisfied if* r_1 *is much larger than* r_2.

Discussion The symmetric Nash bargaining model that we have discussed in the paper can be easily extended to the case of asymmetric Nash bargaining. The asymmetric Nash solution will specify risk-sharing rules $\hat{s}(Y)$, which solves the following problem:

$$\max_{s(Y)} \left((Eu_1(s(\tilde{Y})) - Eu_1(\tilde{y}_1))^\alpha \cdot (Eu_2(\tilde{Y} - s(\tilde{Y})) - Eu_2(\tilde{y}_2)) \right)^{1-\alpha},$$

where the parameter α measures the bargaining power of each bargainer. A higher α means that bargainer 1 has higher bargaining power. A natural question is how bargainer 1's bargaining power α affects the property of comparative statistics. As α increases, bargainer 1 obtains most of the pie. Hence, the net surplus effect caused by an increase in bargainer 2's risk aversion becomes more relevant, while the bargaining power effect caused by an increase in bargainer 2's risk aversion becomes less relevant. In the extreme case where $\alpha \to 1$, only the net surplus effect exists; hence, whether bargainer 1 is betters offer depends only on whether or not an increase in bargainer 2's risk aversion increases the net surplus.

4 Applications

4.1 Bargaining Over Insurance Contracts

In this section, we apply our model to study insurance contracts reached through bargaining. Although Kihlstrom and Roth (1982) have already used this model, they consider only the case with a risk-neutral insurer. The assumption of a risk-neutral insurer is appropriate if the insurer insures many risks independent of the one being analyzed and, hence, diversifies these risks. In other interesting situations, however, the assumption of insurer risk neutrality can not be justified (Kihlstrom and Roth 1982). In this section we apply our basic model and provide a simple method to reconsider their situation but with a risk-averse insurer

Consider a situation with two individuals: a client and an insurer. Both the insurer and the client are risk-averse and have ISHARA utility functions: $u_i(c) = \frac{(c-\theta_i)^{1-\sigma}}{1-\sigma}$, $i = I, C$, where I, C represent insurer and client.

The client faces a possible financial loss. His wealth is a binary random variable:

$$\tilde{w}_C = \begin{cases} w_C > 0 \text{ with probability } v \\ w_C - L > 0 \text{ with probability } 1 - v \end{cases}.$$

The insurer's wealth is w_I and he is not faced with the possibility of any exogenous losses. Assume that the insurer has sufficient wealth to provide complete coverage in any case.

The insurer agrees to insure the client and bear some of the burden of the client's loss in the event that such a loss occurs. His wealth is

$$x_{Il} = w_I - A$$

if the loss occurs and

$$x_{In} = w_I + p$$

if the loss does not occur.

With this insurance contract in force, the client's wealth is

$$x_{Cl} = w_C - L + A$$

if the loss occurs and

$$x_{Cn} = w_C - p.$$

if the loss does not occur.

Let's first consider the case of a risk-neutral insurer and a risk-averse client. In a competitive insurance market, the client is completely insured, and the insurer's expected wealth is equal to w_I. The competitive equilibrium contract (A, p) is unchanged by an increase in the client's risk aversion, and is determined by the following two equations

$$L - A = p,$$

$$vp - (1 - v) A = 0.$$

Now we assume that the insurance contracts are reached through Nash bargaining. Pareto optimality of the Nash solution requires that the risk-neutral bear all the risks. That is, the client is completely insured. Thus, the total surplus C is given by

$$C = w_C + w_I - (1 - v) L,$$

regardless of the degree of the client's risk aversion.

The client's disagreement payoff is $C_2^d \triangleq u_C^{-1}(Eu_C(\tilde{w}_C))$ in monetary terms. As the client becomes more risk-averse, C_2^d decreases. Thus, an increase in the client's risk aversion increases the net certainty equivalent that the insurer and the client are bargaining over because it does not change the total certainty equivalent, but reduces the client's disagreement certainty equivalent. It follows from proposition 1 that the insurer is better off, which is the result of theorem 4.1 in Kihlstrom and Roth (1982).

Moreover, that the insurer is better off means that $(1 - v) p - vA$ increases. Because the client is completely insured, we have $L - A = p$. It follows immediately that p increases and A decreases: A more risk-averse client pays a higher premium and receives less coverage of his potential loss.

Now we turn to the case in which both the insurer and the client are risk-averse. The total certainty equivalent is $C = w^{-1}(Ew(\tilde{w}_C + w_I))$, where w is the representative's utility function, with $w = \frac{(c-\theta_C-\theta_I)^{1-\sigma}}{1-\sigma}$. The disagreement payoff of the client is equal to $C_2^d \triangleq u_C^{-1}(Eu_C(\tilde{w}_C))$, where $u_C = \frac{(c-\theta_C)^{1-\sigma}}{1-\sigma}$.

Lemma 3 *The net certainty equivalent increases as the client becomes more risk-averse.*

Proof. We need to prove

$$\frac{\partial NC}{\partial \theta_C} = \frac{\partial C}{\partial \theta_C} - \frac{\partial C_2^d}{\partial \theta_C} \geq 0.$$

Using the specific formula of u_C and w, we know that

$$C_2^d = \left[E\left(\tilde{w}_C - \theta_C\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} + \theta_C$$

and that

$$C = \left[E\left(\tilde{w}_C + w_I - \theta_I - \theta_C\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} + \theta_I + \theta_C.$$

Notice that the insurer has sufficient wealth and, hence, $w_I - \theta_I > 0$. The above two equations imply that we will be done if we can prove $\frac{\partial^2 C}{\partial \theta_C \partial \theta_I} \leq 0$. From the above equation, we have

$$\frac{\partial C}{\partial \theta_C} = -\left[E\left(\tilde{w}_C + w_I - \theta_I - \theta_C\right)^{1-\sigma}\right]^{\frac{\sigma}{1-\sigma}} E\left(\tilde{w}_C + w_I - \theta_I - \theta_C\right)^{-\sigma} + 1,$$

and, therefore,

$$\begin{aligned}
\frac{\partial^2 C}{\partial \theta_C \partial \theta_I} &= \sigma\Big\{\left[E\left(\tilde{w}_C + w_I - \theta_I - \theta_C\right)^{1-\sigma}\right]^{\frac{2\sigma-1}{1-\sigma}} \left(E\left(\tilde{w}_C + w_I - \theta_I - \theta_C\right)^{-\sigma}\right)^2 \\
&\quad - \left[E\left(\tilde{w}_C + w_I - \theta_I - \theta_C\right)^{1-\sigma}\right]^{\frac{\sigma}{1-\sigma}} E\left(\tilde{w}_C + w_I - \theta_I - \theta_C\right)^{-\sigma-1}\Big\} \\
&= \sigma\left[E\left(\tilde{w}_C + w_I - \theta_I - \theta_C\right)^{1-\sigma}\right]^{\frac{2\sigma-1}{1-\sigma}} \Big\{\left(E\left(\tilde{w}_C + w_I - \theta_I - \theta_C\right)^{-\sigma}\right)^2 \\
&\quad - E\left(\tilde{w}_C + w_I - \theta_I - \theta_C\right)^{1-\sigma} E\left(\tilde{w}_C + w_I - \theta_I - \theta_C\right)^{-\sigma-1}\Big\} \\
&\leq 0,
\end{aligned}$$

where the last inequality holds as a direct application of Cauchy-Schwarz inequality. ■

Notice that an increase in the client's degree of risk aversion reduces both the total surplus and the client's certainty equivalent of disagreement. In the case of no disagreement, the certainty equivalent C_2^d is reduced significantly because the client alone bear all the risk. In the case of agreement, however, the total certainty equivalent C is reduced only slightly because the insurer share some of the risk. As a result, the reduced amount of C is much less than the reduced amount of C_2^d, and, therefore, the net certainty equivalent increases.

The above lemma, together with Proposition 1, immediately gives the following proposition:

Proposition 2 *The insurer, whether he is risk-neutral or risk-averse, benefits as the client becomes more risk-averse.*

Calculating the Bargained Insurance Contract. Now we illustrate how we can calculate the bargained contract (A, p) from the transformed problem. First, from equation (1), we can calculate the exact net certainty equivalent that the insurer gets ($x^{\#}$) and the client gets ($NC - x^{\#}$). Then, the contract (A, p) can be calculated from the definition of certainty equivalent, which is given by the following two equations:

$$vu_C\left(w_C - p\right) + (1 - v)\, u_C\left(w_C - L + A\right) = u_C\left(C_2^d + NC - x^{\#}\right),$$
$$vu_I\left(w_I + p\right) + (1 - v)\, u_I\left(w_I - A\right) = u_I\left(w_I + x^{\#}\right).$$

4.2 Bargaining Over the Incentive Contract

Standard principal-agent models often assume that the principal offers "take-it-or-leave-it" contracts to the agent. Consequently, the principal obtains all the surplus of the transaction. A direct result is that the principal suffers from an increase in the degree of the agent's risk aversion, because the cost of providing higher incentive increases as the agent becomes more risk-averse.

However, in many real-world situations, both parties hold some bargaining power and, thus, the contracting involves bargaining. For example, many labour market situations are characterized by bargaining between workers and firms (Demougin and Helm 2006). We will prove in this section that bargaining will significantly change the property of comparative statistics. In particular, we will show that the principal may benefit if the agent becomes more risk-averse.

Consider the case in which a risk-neutral principal is bargaining with a risk-averse agent over an incentive contract. The principal hires the agent to produce output. The agent has CARA utility function with absolute risk-averse coefficient r: $u\left(x\right) = \frac{1-\exp(-rx)}{r}$. The agent can exert costly effort to increase output. The output is

$$y = e + \varepsilon,$$

where $\varepsilon \sim N\left(0, \sigma^2\right)$, with σ^2 representing the riskiness involved in the production process and, e representing the effort exerted by the agent. The effort cost is $c\left(e\right) = \frac{e^2}{2}$.

Contract. The effort is not observable. The only observable and contractible variable is the output y. Assume that the contract that the two parties are bargaining over is linear:

$$w = w_0 + \alpha y,$$

where w_0 is the fixed salary and α is the power of incentive.

The timing is as follows. First, the two parties engage in a Nash bargaining process and bargain over the contract (w_0, α). If they reach no agreement, then the game is over and both of them get nothing. If they sign a contract, then the agent chooses his effort. Finally, output is realized and the contract is executed.

Given the contract, the agent chooses e to maximize his certainty equivalent

$$C_A = w_0 + \alpha e - \frac{r\alpha^2\sigma^2}{2} - \frac{e^2}{2}.$$

F.O.C with respect to e gives the following incentive-compatible condition

$$IC : e = \alpha.$$

For contract (w_0, α), the total certainty equivalent of the principal and the agent is

$$C = e - \frac{e^2}{2} - \frac{r\alpha^2\sigma^2}{2}.$$

Substituting the IC condition into the expression of C and C_A, we obtain $C(\alpha) = \alpha - \frac{(1+r\sigma^2)\alpha^2}{2}$ as a function of α and $C_A(\alpha, w_0) = w_0 + \frac{(1-r\sigma^2)\alpha^2}{2}$ as a function of α and w_0. Notice that both the principal and the agent get nothing if no agreement is reached. Hence, the Nash Bargaining solution is given by the following problem

$$\max_{w_0,\alpha} \left(C(\alpha) - C_A(\alpha, w_0)\right) u\left(C_A(\alpha, w_0)\right).$$

The solution of Nash bargaining implies that two parties will choose α to maximize the total certainty equivalent $C(\alpha)$. Otherwise, suppose that the solution is (w_0', α'), while there exists α^* such that $C(\alpha^*) > C(\alpha')$. Then one can choose a proper w_0^* such that $C_A(\alpha', w_0') = C_A(\alpha^*, w_0^*)$. Obviously, (w_0^*, α^*) gives a higher value of $(C(\alpha) - C_A(\alpha, w_0))\, u(C_A(\alpha, w_0))$, contradicting that (w_0', α') is the Nash solution.

The first-order condition of $C'(\alpha) = 0$ immediately gives

$$\alpha^* = \frac{1}{1 + r\sigma^2}.$$

The net certainty equivalent is equal to the total surplus and is given by

$$\begin{aligned} NC = C &= \frac{1}{1+r\sigma^2} - \frac{1}{2}\left(\frac{1}{1+r\sigma^2}\right)^2 - \frac{r\sigma^2}{2}\left(\frac{1}{1+r\sigma^2}\right)^2 \\ &= \frac{1}{2}\frac{1}{1+r\sigma^2}. \end{aligned}$$

Proposition 3 *Compared to the case in which the principal has all the bargaining power, the bargaining model predicts the same power of incentive (α) and, hence, the same total surplus.*

The existing literature on bargaining contracts between principals and agents often assumes a risk-neutral agent with limited liability (Pitchford 1998, Balkenborg 2001, Demougin and Helm 2006). The main result is that the bargaining model and the take-it-or-leave-it model predict different incentives. The above proposition is in contrast with this result and provides an example of when bargaining does not have a real effect. However, as we will show immediately, the principal's preference over the agent's degree of risk aversion is quite different from the take-it-or-leave-it model.

The above analysis shows that the bargaining model can be viewed as if the principal and the agent were bargaining over a total surplus $C = \frac{1}{2}\frac{1}{1+r\sigma^2}$, with the outside option normalized to zero. Hence, we can rewrite the problem as:

$$\max_x xu\left(C - x\right)$$

The first-order condition gives:

$$\frac{1 - \exp\left(-r\left(C - x\right)\right)}{r} - x\exp\left(-r\left(C - x\right)\right) = 0,$$

from which we get

$$x = \frac{1}{r}\left(\exp\left(r\left(C - x\right)\right) - 1\right),$$

where $C = \frac{1}{2}\frac{1}{1+r\sigma^2}$. Define $L = \frac{1}{r}\left(\exp\left(r\left(C - x\right)\right) - 1\right)$; then, we know that $\frac{\partial x}{\partial r} \geq 0$ iff $\frac{dL}{dr} \geq 0$.

$$\begin{aligned}\frac{dL}{dr} &= \frac{\partial L}{\partial r} + \frac{\partial L}{\partial C}\frac{\partial C}{\partial r} \\ &= \frac{1}{r^2}\left[1 + r\left(C - x\right)\exp\left(r\left(C - x\right)\right) - \exp\left(r\left(C - x\right)\right)\right] \\ &\quad - \exp\left(r\left(C - x\right)\right)\left[\frac{1}{2}\frac{\sigma^2}{\left(1 + r\sigma^2\right)^2}\right]\end{aligned}$$

Obviously, for σ^2 close to zero, $\frac{dL}{dr}$ is strictly positive. Hence, the principal benefits from an increase in the agent's risk aversion.

Proposition 4 *The principal may benefit from or be hurt by an increase in the agent's degree of risk aversion. Specially, he benefits from an increase in the agent's degree of risk aversion if the production process is sufficiently riskless.*

An increase in the agent's degree of risk aversion has two effects. On the one hand, it reduces the total surplus, which hurts the principal. On the other hand, the agent's bargaining power becomes weaker as he becomes more risk-averse, which benefits the principal. For a sufficiently riskless production process, the first effect is dominated by the second one, leading to a higher utility for the principal.

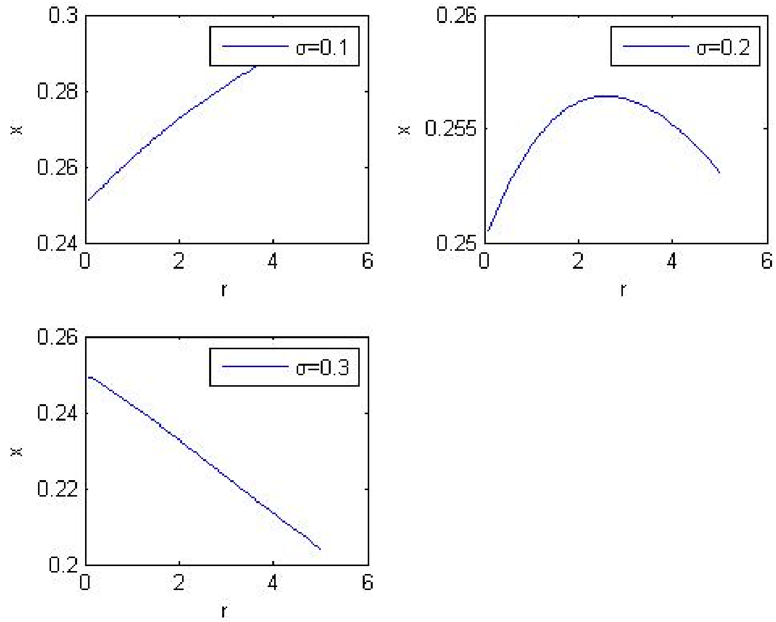

Figure 1

Figure 1 illustrates how the principal's payoff x varies with the agent's degree of risk aversion, given different σ. We can see that for a small value of σ, the principal's payoff is increasing in the agent's degree of risk aversion; for a large value of σ, the principal's payoff is decreasing in the agent's degree of risk aversion; for a middle value of σ, the principal's payoff is first increasing and then decreasing in the agent's degree of risk aversion.

5 Concluding Remarks

This paper builds a simple Nash bargaining model with uncertainty. In particular, we identify the two effects of a change in one bargainer's degree of risk aversion: the bargaining power effect and the net surplus effect. An increase in one bargainer's degree of risk aversion reduces his bargaining power, while, at the same time, it changes the net surplus. Whether this benefits his opponent depends on which effect dominates.

The simplicity of our model allows us to apply it in many situations. In an application to bargaining over insurance contracts, we show that the result in Kihlstrom and Roth (1982), which states that a risk-neutral insurer is better off if the insured becomes more risk-averse, is also robust for the case of the risk-averse insurer. Applying our model to the situation of bargaining over incentive contracts, à la Holmstrom and Milgrom (1987), we show that the principal may benefit as the agent becomes more risk-averse, in contrast to the prediction of the take-it-or-leave-it model.

We believe that our model has many other applications, because many real-world bargaining games involve uncertainty. For example, our model can be applied in situations in which agents form small groups (marriage, partnership, etc.) for the purpose of risk sharing. In India, families find suitable men for their daughters from distant villages to reduce correlation in climatic and production shocks. A primary concern is about the compositions of such risk-sharing partnerships—i.e., whether agents in the group have similar or dissimilar risk preferences. An important issue in this literature is the conflict between theory and empirical/experimental evidences. Theoretical matching models predict negative assortative matching (Chiappori and Reny 2006; Legros and Newman 2007; Schulhofer-Wohl 2006). This result is, however, not consistent with the empirical and experimental literature (Lam 1988; Charles and Hurst 2003; Di Cagno et al 2012).

One possibility to resolve this conflict is to relax the assumption that the risk-sharing rule is determined by the competitive market in the theoretical models. Instead, one can assume that agents share their joint risky income through Nash bargaining. If we can show that an agent suffers if his partner becomes more risk-averse under some conditions, then the resulting matching will be positively assortative. The most risk averse agent will make a proposal to the most risk averse partner, who is happy to accept the offer. As a result, agents in the group have similar preferences, which is consistent with empirical and experimental evidence.

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